THEORY OF PROBABILITY

UNIT - 1

Elementary Probability Theory

The probability theory has been developed to estimate the proportion of Uncertainity of experimental result when experiment conducted at random. probability means the chance of happening. if we express the chance of happening on an event in numerical terms it is called the probability of that event. in other words probability of an event can be defined as the numerical expression of likelyhood of occurance of the events.

Random experiment

An experiment that can be repeated under Same condition and the outcomes of which cannot be predicted in any repetitions are called random experiments

eg:- Tossing a coin, throwing a die . . .

Essential features of a Random experiments are

- it has more than one outcome.
- the experiment is repeatable.

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- The outcomes in any repetition are unpredictable for the probability of an already defined event must remain in the same in each experiment.

Sample Space

The Set of all possible outcomes of random experiments is called Sample Space it is usually denoted by S. Ω.

The elements of Sample Space are called Sample points.

eg:- • Tossing a coin, S = {H, T}.

• Throwing a die S = {1, 2, 3, 4, 5, 6}.

• Throwing 2 die Simultaneously

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }.

• in a random of two coin

S = { (H,H), (H,T), (T,T), (T,H) }.

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generally in a random toss of N coins,
no of Sample space $n(S) = 2^N$

generally in a random toss of N dices,
no of Sample Space $n(S) = 6^N$

Events

In any Subsets of Sample Space which contain one or more Sample point is called event.
eg:- consider an experiment of throwing a dice once. A be the event of getting odd number and B be the event of getting even numbers.
hence $S = \{1, 2, 3, 4, 5, 6\}$. $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$.
$\therefore$ A and B are events as they are Subsets of S.
if an event contains only a Single Sample points as its element. it is called Simple event.
if an event having 2 or more Sample points as its element. then it is called a compound event.
eg:- let $A = \{1\}$. $B = \{2, 3, 4\}$.

Mutually exclusive events

events are Said to be mutually exclusive if the happening of any one event prevents the happening of all the other.

eg:- if a coin is tossed either head or turns up or tail can turn up. both cannot come up at the same time.
ie here $S = \{H, T\}$. let $A = \{H\}$, $B = \{T\}$.
$\therefore A \cap B = \phi$.

eg:- In throwing a die, all the six faces numbered 1-6 are mutually exclusive events. Since if any one of these faces comes, the possibility of others in the same trial is ruled out.

Exhaustive events

A group of events are said to be exhaustive when it includes all possible outcomes of the random experiment. In other words, if the union of events considered is the whole sample space. Then the event are called exhaustive events

eg:- if a coin is tossed. let A be the event of getting head and B be the event of getting tail. here $S = \{H, T\}$. $A = \{H\}$
$B = \{T\}$. $\because A \cup B = \{H, T\} = S$
$\therefore$ A and B are exhaustive events.

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eg:- if a die is tossed. Let A be event of getting even numbers and B be the event of getting odd numbers $A = \{2,4,6\}$
$B = \{1,3,5\}$. $A \cup B = \{1,3,4,5,6\}$.
~~∴ AOB are~~ ∴ A and B are exhaustive events.

Equally Likely Events

when all the events under consideration are having equal chance to occur in any repetition of random experiment. The events are called the equally likely events. ie outcomes of a trial are said to be equally likely, if there is no reason to expect to ~~or~~ ~~ne~~ one in the preference of other

eg:- Tossing an unbiased coin, Head or tail are equally likely event.

eg:- In throwing an unbiased die, all the six faces are equally likely to come.

Favourable cases.

The outcomes which entail the happening of the event are called favourable cases for that event.

eg:- Consider an experiment of throwing a die.

Let A be the event of getting an even number
A = {2, 4, 6}. So the occurrence of 2 or 4 or 6.
are the favourable events.

Independent Events

Several events are said to be independent if the happening (or non happening) of an event is not effected by the occurrance of any of the remaining event.

eg:- In tossing an unbiased coin, the event of getting an head in the 1st throw is independent of getting a head in the 2nd, 3rd and subsequent toss

eg:- If we draw a card from a pack of well shuffeled card, replace it before drawing 2nd card. The result of 2nd draw is independant of 1st draw.
But however if the 1st card drawn is not replaced then the 2nd draw is dependent of the 1st draw.

Algebra of events

for events A, B, C

1) $A \cup B = \{x \in S : x \in A \text{ or } x \in B\}$
2) $A \cap B = \{x \in S : x \in A \text{ and } x \in B\}$
3) $A - B = \{x \in S : x \in A \text{ and } x \notin B\}$
4) $B - A = \{x \in S : x \in B \text{ and } x \notin A\}$
5) $A' = \{x \in S : x \notin A\}$.
6) $A \subset B$ = for every $x \in A$, $x \in B$.
7) A and B ~~am~~ are disjoint $\Rightarrow A \cap B = \phi$.
8) $A \cup B$ can be denoted as $A + B$. if A and B are disjoint

Partition Of a Sample Space

A partition of Sample space is a Set of mutually exclusive events that together cover the entire Sample Space. This means that the events in a partition cannot occur Simultaneously (ie they are mutually exclusive). and atleast one of the event must occur everytime an experiment is performed (ie they are collectively exhaustive)

Mathematically if S is a Sample Space and $E_1, E_2, \dots E_n$ is an partition of S. Then the following 2 condition must be Satisfied.

1) The events $E_1, E_2, E_3, \dots E_n$ are mutually exclusive

This means that for any 2 distinct events E_i and E_j in the partition, their intersection is empty. in other words $E_i \cap E_j = \phi \ \forall i \neq j$

2) The Union of all the events in the partition is the entire Sample Space. ie $E_1 \cup E_2 \cup \ldots \cup E_n = S$

This ensures that every outcome in the Sample Space is included exactly in one of the events in partition.

eg:- Consider an experiment of throwing a die

$S = \{1, 2, 3, 4, 5, 6\}$.

below are the some of the partition. of the Sample space.

$A_1 = \{1, 2, 3\}$. $A_2 = \{4, 5, 6\}$.

$B_1 = \{1\}$ $B_2 = \{2, 3, 4, 5\}$ $B_3 = \{6\}$.

$C_1 = \{1, 2\}$ $C_2 = \{3, 4, 5, 6\}$.

Theory Of Probability

1) Classical definition of Probability (Mathematical)
2) Frequency definition of Probability (Statistical)
3) Axiomatic definition of Probability (Modern)

Classical definition of Probability

If a random experiment result in n. exhaustive, mutually exclusive and equally likely outcomes, out of which m are favourable to the occurence of an event E, then the probability P of the occurence of E is given by

$$P = P(E) = \frac{\text{No of favourable Cases.}}{\text{Total no of Cases}} = \frac{m}{n}$$

Remark.

Since $m \geq 0$, $n > 0$, $m \leq n$, Then $P(E) \geq 0$ and. $P(E) \leq 1 \Rightarrow 0 \leq P(E) \leq 1$

The probability of non occurance of an event is denoted ~~to~~ as $P(\bar{E})$ or $P(E')$.

if n is the total no of possible outcomes out of which m is the no of favourable cases. then n-m will the no of ~~f~~ unfavourable cases.

Thus $P(E') = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(E)$.

$\Rightarrow P(E') + P(E) = 1$

Note.

- if A is an impossible event $P(A) = 0$
- if A is a Sure event ~~$P(A) = 0$~~ $P(A) = 1$
- if A is a random event $0 < P(A) < 1$

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Limitations of Classical definition

- The definition of classical probability breaks down into following cases. if the ~~[illegible]~~ various outcomes of the trial are not equally likely or equally probable.

eg:- The probability that a candidate will pass in a certain test is not 50%. Since the 2 possible outcomes success and failure are not equally likely.

- if the exhaustive ~~[illegible]~~ no of cases in a trial is infinite

eg:- probability of even number =

$P(E) = \frac{\infty}{\infty}$, = not defined by definition

but probability of even numbers is equal to $1/2$.

In both the cases, we cannot use classical definition.

Q) Consider an experiment of picking a marble from a bag containing 5 red marbles and 3 blue marbles.

Ans) Let E be the event of picking a red marble. Then find the probability of E

Ans) Here Total no of Marbles is 8 (5+3)
favourable outcomes : 5 red marbles.

$$\therefore P(E) = \frac{\text{Total no of favourable cases}}{\text{Total cases}} = \frac{5}{8}$$

Q) Consider an experiment of rolling a 6 sided die. Let E be the event of getting a number greater than 4. find the probability of E.

Ans) Here the possible outcomes are 1, 2, 3, 4, 5, 6. So total no of possible outcomes are 6. E is the event of getting a number greater than 4

$\Rightarrow$ favourable cases = 5 & 6.

$\therefore$ Probability of getting a number greater than 4

$$P(E) = \frac{\text{No of favourable cases}}{\text{Total Cases}} = \frac{2}{6} = \frac{1}{3} //$$

Q) Consider an experiment of drawing one card from standard deck of 52 playing cards. find the probability of drawing a red.

Ans) ~~Here~~ In a deck of 52 cards 13 cards are.
13 are , 13 are , and 13 are
All and are red and
all and are black.

Total red card = 26
⇒ favourable cases = 26
∴ The probability of drawing a red card

$$= \frac{\text{No of favourable cases}}{\text{Total No of cases}} = \frac{26}{52} = \frac{1}{2}$$

Q) Three coins are tossed. What is the probability of getting
1) all heads
2) exactly one head
3) exactly two head
4) atleast one head
5) atleast two head
6) atmost one head
7) atmost two head
8) no head

Ans $S = \{HHH, HTT, THT, TTH, HTH, HHT, THH, TTT\}$

(1) All are head
favourable cases = HHH
P(all are head) = 1/8

(2) Exactly one head
favourable cases = HTT, THT, TTH

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P(exactly one head) = 3/8

(3) exactly two head.

favourable cases = HTH, HHT, THH

P(exactly two head) = 3/8

(4) Atleast one head.

favourable cases = HHH, HTT, THT, TTH, HTH, HHT, THH.

P(atleast one head) = 7/8.

(5) Atleast two head.

favourable cases = HHH, HTH, HHT, THH.

P(atleast two head) = 4/8 = 1/2.

(6) Atmost one head

favourable case = HTT, THT, TTH, TTT

P(atmost one head) = 4/8 = 1/2.

(7) Atmost two head.

favourable cases = HTT, THT, TTH, HTH, HHT, THH, TTT

P(atmost two head) = 7/8

(8) No head.

favourable Cases = TTT

P(no head) = 1/8

Q). What is the probability of a leap year selected contains 53 sundays.

Ans) In a leapyear there are 366 days consisting of 52 weeks + 2 more days. The following are the

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possible combinations of these days.

① Mon. tue ② tue wed ③ wed thur ④ Thur - fri
⑤ Fri - sat ⑥ Sat Sun ⑦ Sun Mon.

for getting 53 Sundays in a leap year out of the two days so obtained. one should be Sunday. There are two cases favourable for getting a Sunday out of these 7 cases.

⇒ ∴ required Probability = 2/7

possible combinations of these days.
① Mon. tue ② tue wed ③ wed thur ④ thur. fri
⑤ Fri - sat ⑥ Sat Sun ⑦ Sun mon.
for getting 53 sundays in a leap year out of the two days so obtained. one should be Sunday. There are two cases favourable for getting a Sunday out of these 7 cases.
⇒ ∴ required probability = 2/7

6 Q) Two unbiased dias are thrown find the probability that
a) both the dias show the same number.
b) The 1st die shows six
c) the total ~~of the~~ sum of the numbers on the dice is 8.
d) the total sum of the numbers on the dice is greater than 8.
e) the total sum of the numbers on the dice is 13.
f) the total sum of the number on the dice is any number from 2 to 12, both inclusive.
Ans)

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S = { (1,1) (2,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }.

a) Probability that both the dice show the Same number : Favorable Cases = (1,1) (2,2) (3,3) (4,4) (5,5) (6,6)

No of favourable Cases = 6.

Probability $= \frac{6}{36} = \frac{1}{6}$.

b) Probability that 1st die shows Six.

Favourable cases = (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

No of favourable cases = 6

Probability $= \frac{6}{36} = 1/6$ //

c) The total Sum of the numbers on the dice is 8.

Favourable cases = (2,6) (3,5) (4,4) (5,3) (6,2)

No of favourable cases = 5.

Probability $= \frac{5}{36}$.

d) The total Sum of the numbers on the dice is greater than 8

favourable cases = (3,6) (4,5) (4,6) (5,4) (5,5) (5,6) (6,3) (6,4) (6,5) (6,6)

No of favourable cases = 10

Probability $= \frac{10}{36}$ //

e) The total sum of numbers on the dice is 13

Favourable case = { }

No of favourable cases = 0

Probability $= \frac{0}{36} = \underline{\underline{0}}$.

f). The total sum of the numbers on the dice is any number from 2 to 12. both inclusive

No of favourable cases = 36.

Probability $= \frac{36}{36} = \underline{\underline{1}}$.

7Q) 4 cards can be drawn at random from a pack of 52 cards. find the probability of

a) they are king, queen a jack and a ace.

b) 2 are king and 2 are queen.

c) 2 are black and 2 are red.

d) 2 cards of heart and 2 of diamond.

Ans). 4 cards can be drawn from a pack of well shuffeled. 52 cards in $52C_4$ ways, which give the exhaustive no of cards.

a) they are a king, a queen, a jack and a ace.

One king can be drawn from 4 kings in $4C_1$ ways. Similarly one queen, one jack, and

and an ace, each can be drawn in $4C_1$ ways.
Since any one of the ways of drawing a king can be associated with any one of the ways of drawing a queen, jack and ace.
The favourable no of cases $= 4C_1 \times 4C_1 \times 4C_1 \times 4C_1$.
~~Proof~~ Hence required probability $= \dfrac{\text{No of favourable cases}}{\text{Total No of cases}}$

$$= \frac{4C_1 \times 4C_1 \times 4C_1 \times 4C_1}{52C_4}$$

$$= \frac{4 \times 4 \times 4 \times 4}{\frac{\overset{13}{\cancel{52}} \times \overset{17}{\cancel{51}} \times \overset{25}{\cancel{50}} \times 49}{1 \times 2 \times 3 \times 4}}$$

$$= \frac{256}{270725}$$

b) Probability of drawing 2 queen and 2 king
2 kings $\Rightarrow 4C_2$ ways. Similarly $\Rightarrow 4C_2$ ways.
Favourable cases $= 4C_2 \times 4C_2$
Hence Required Probability $= \dfrac{4C_2 \times 4C_2}{52C_4} = \dfrac{16}{270725}$

c) 2 are black and 2 are red.
2 black $\Rightarrow 26C_2$. 2 red $\Rightarrow 26C_2$.
Favourable cases $= 26C_2 \times 26C_2$
Required Probability $= \dfrac{26C_2 \times 26C_2}{52C_4} = \dfrac{325}{270725}$

d) 2 cards of heart and 2 of diamond

2 heart $\rightarrow 13C_2$ 2 diamond $\rightarrow 13C_2$

Total favourable cases - $13C_2 \times 13C_2$

Required Probability $= \frac{13C_2 \times 13C_2}{52C_4} = \frac{6084}{270725}$

8Q) A Urn contains 6 white, 4 red, and 9 black balls if 3 balls are drawn at random. find the probability that

a) two of the balls are white

b) one is of each colour.

c) none is red.

d) atleast one is white

Ans) The total no of balls is 19, so 3 balls can be drawn in $19C_3$ ways.

a) Two of the balls are white

2 White $\rightarrow 6C_2$. 1 Non white $\Rightarrow 13C_1$

Total favourable cases = $6C_2 \times 13C_1$

Required Probability = $\frac{\text{Favourable}}{\text{Total}} = \frac{6C_2 \times 13C_1}{19C_3}$

$= \frac{\cancel{15\times5}}{\cancel{165}} = \frac{\cancel{75}}{\cancel{165}} // = \frac{}{969}$

b) One is of each colour

1 white $\rightarrow 6C_1$ 1 red $\rightarrow 4C_1$ 1 black $\rightarrow 9C_1$

No of favourable cases = $6C_1 \times 4C_1 \times 9C_1$

Probability $= \dfrac{6C_1 \times 4C_1 \times 9C_1}{19C_3}$

$= \dfrac{6 \times 4 \times 9}{\cancel{165}\ 969} = \dfrac{\cancel{54}}{\cancel{165}}$

$= \dfrac{216}{969}$.

c) None is red.

~~3 Red => 4C3~~. 3 Non red => $15C_3$ ways

~~Probability of getting all red = $\frac{4C_3}{11C_3} = \frac{4}{165}$.~~

No of favourable cases = $15C_3$.

Probability of getting no red = $\dfrac{15C_3}{19C_3} = \dfrac{\cancel{35}}{\cancel{165}}\ \dfrac{\cancel{286}\ 455}{969}$

d) Atleast one is white

3 non white => $13C_3$ ways.

Probability of getting no white $= \dfrac{13C_3}{19C_3} = \dfrac{\cancel{10}}{\cancel{165}} . \dfrac{286}{969}$.

$= \cancel{683/683}$

Probability of getting atleast one white = 1 − P of getting no white

$= 1 - \dfrac{286}{969}$

$= \dfrac{\cancel{165 - 10}}{\cancel{165}}\ \dfrac{969 - 286}{969}$

$= \dfrac{\cancel{155}}{\cancel{165}} = \dfrac{683}{969}$

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9Q) A Commitee of 4 people is to be appointed from
3 officers of the production department.
4 officers of the purchase department.
2 of the sales department and
1 of chartered accounting.
find the probability of forming the commitee in the following manner.
1) There must be one from each category.
2) it should have atleast one from the purchase department.
3) The chartered accountant must be in the commitee.

1) one from each depart => $3C_1 \times 4C_1 \times 2C_1 \times 1C_1$

P(one per from each category) : $\frac{3C_1 \times 4C_1 \times 2C_1 \times 1C_1}{10C_4}$

2) ~~favourable cases:~~
None from purchase department -> $6C_4$
Probability of none one purchase department = $\frac{6C_4}{10C_4}$

$= \frac{15}{210}$

probability of atleast one = 1 − P(none from purchase).

$$= 1 - \frac{15}{210}$$

$$= \frac{210-15}{210} = \frac{\cancel{180}\ 195}{210}$$

3). favourable cases => $1C_1 \times 9C_3$

Probability $= \dfrac{1C_1 \times 9C_3}{10C_4}$

$$= \frac{1 \times \frac{9 \times 8 \times 7}{1 \times 2 \times 3}}{210} = \frac{84}{210}$$

Q) A man is dealt with 4 spade cards from an ordinary pack of 52 cards. if He is given 3 more cards find the probability P that atleast one of the additional card is also spade.

52 cards ⟶ 48 + $\boxed{\text{4 cards}}$ => spade.

$$48 \begin{cases} \to 13 \text{ heart} \\ \to 13 \text{ diamond} \\ \to 13 \text{ club} \\ \to 9 \text{ spade} \end{cases}$$

$\underbrace{13 \text{ heart},\ 13 \text{ diamond},\ 13 \text{ club}}_{39 \text{ non spade}}$

P(atleast 1 spade) = 1 −

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After the man has dealt 4 spade cards from a ordinary pack of 52 cards there are 52-4 i.e = 48 cards in the packet. out of which 9 are spade cards and 39 are non spade cards.

Since 3 more cards can be dealt to the same man. out of the 48 cards in $48C_3$ ways, the exhaustive number of outcomes = $48C_3$

if none of these 3 additional cards is a spade card, then the three additional cards must be drawn out of the 39 non spade cards, which can be done in $39C_3$ ways.

∴ The probability that none of the three additional cards dealt to the man is not a spade card is given by $\frac{39C_3}{48C_3}$

hence The probability (P) that atleast one of the three additional cards is a spade card is given by $1 -$ P(none of the additional 3 cards in spade)

$$= 1 - \frac{39C_3}{48C_3}.$$

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11Q) If the letters of the word 'REGULATIONS' be arranged at random, what is the chance there will be exactly 4 letters between R and E.

Ans) The word regulation consist of 11 letters, the two letters R and E can occupy $^{11}P_2$ i.e 11×10 110 positions. The number of ways in which there will be exactly four letters R and E are enumerated below.

1) R in 1st place E is 6th place
2) R in 2nd E in 7
3) R in 3rd E in 8
4) R in 4th E in 9
5) R in 5th E in 10
6) R in 6th E in 11th place.

Since R and E can interchange their positions, the required no of favourable cases = $2 \times 6 = 12$.

$\therefore$ The required probability = $\frac{12}{110} = \frac{6}{55}$ //

12Q) 25 books are placed at random in a shelf. find the probability that a particular pair of books shall be

•) always together
•) Never together

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Since 25 books can be arranged themselves in 25! ways, the exhaustive no of cases is 25!

1° a) Let us now regard that the two particular books are tagged together so that we shall regard them as a single book. Thus now we have (25 - 1) = 24 book which can be arranged among themselves in 24! ways. But the 2 books which are tagged together can be arranged among themselves in 2! ways. Hence associating these two operations. The no of favourable cases for getting a particular pair of books always together is $24! \times 2!$.

$\therefore$ Required Probability $= \dfrac{24! \times 2!}{25!}$

b) ~~Probability of~~

P(a particular pair of books should not be together) = 1 - P(a particular book should be together)

$= 1 - \dfrac{24! \times 2!}{25!}$

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Q) n persons are seated on n chair at round table. find the probability that two specified persons are sitting next to each other.

Ans) The different ways in which we can arrange n persons seated on n chairs at round table is $(n-1)!$.

ie The total no of ways in which they can be seated $= (n-1)!$

if these two persons are regarded as a single so there are $= (n-1)$ persons. so total ways $= (n-2)!$

Total favourable cases $= 2 \times (n-2)!$

$n=8$ $8P_2$ $\frac{8-1}{7!}$ $6\times5\times4$ $7\times6\times$ $7\times(n-1)!$ $(n-1)(n-2)!$

So Probability $= \dfrac{2 \times (n-2)!}{(n-1)!}$

$= \dfrac{2 \times (n-2)!}{(n-1)(n-2)!}$

$= \dfrac{2}{(n-1)}$

Q) Compare the chances of throwing 4 with one die 8 with 2 dice and 12 with ~~3 with~~ 3 dice

Ans) a) Probability of throwing 4 with 1 die.

There are 6 possible ways in which die can be thrown. and ~~t is~~ of these 1 is favourable to the required event.

$\therefore$ Required probability $P(1) = \frac{1}{6}$.

b) Probability of throwing 8 with 2 die

The exhaustive no of cases with 2 dice is $6^2 = 36$. Now the Sum of '8' can be obtained on the two dice. ~~on~~ in the following ways. (2,6) (6,2) (3,5) (5,3) (4,4) ie 5 cases in all. where 1 and 2nd no in the brackets refer to the no on the 1st and 2nd die respectively.

$\therefore$ Required Probability $P(2) = \frac{5}{36}$.

c) Probability of throwing 12 with 3 die

The exhaustive no of ways with 3 dice is $6^3 = 216$. ~~Now the Sum of~~ '12' To make a throw of 12, The three dice must show the face either (6,1,5) or (6,2,4) or (6,3,3) or (5,2,5) or (5,3,4) or (4,4,4)

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The 1st 2 of these arrangements can occur in $3! = 6$ ways each. The second two ie ~~3rd 4th~~ ie (3rd and 4th) in $\frac{3!}{2!\times1!} = 3$ ways each. The fifth in $3!$ ways and the last one in one way only.

Thus the total no of favourable cases

$= 6+6+3+3+6+1$

$= 25$ ways.

$\therefore$ Required Probability $P(3) = \frac{25}{216}$.

$P_1 : P_2 : P_3 = \frac{1}{6} : \frac{5}{36} : \frac{25}{216}$.

$= 36 : 30 : 25$

$\therefore P_1 : P_2 : P_3 = 36 : 30 : 25$

Q) In a random arrangement of the letters of the word 'COMMERCE' find the probability that all the vowels come together.

Ans) The total no of permutations of the letters of the word 'Commerce' are $\frac{8!}{2!\times2!\times2!}$

because it contains 8 letters of which 2 C, 2 M and

2E. and remaining are all different
The commerce contains 3 vowels i.e OEE
2E being identical. To obtain the total
no of arrangements in which these ~~two~~
three vowels come together, we regard
them as tied together forming only one letter
So that total no of letters in commerce.
may be taken as 6, out of which 2 are
C, 2 are M. and rest distinct.

∴ Their no of arrangements are given by $\frac{6!}{2! \times 2!}$

$\frac{6!\ \cancel{\infty}}{2}$ (OEE)

further the three vowels ~~can be~~ two of
which are identical can be arranged
among themselves in $\frac{3!}{2!}$ ways.

Hence Total no of arrangements favourable
to getting all vowels together $= \frac{6!}{2! \times 2!} \times \frac{3!}{2!}$

∴ Required Probability $- \dfrac{\dfrac{6! \times 3!}{\cancel{2!} \times 2! \times \cancel{2!}}}{\dfrac{8!}{\cancel{2!} \times 2! \times \cancel{2!}}}$

$= \frac{6! \times 3!}{6! \times 7 \times 8} = \frac{3 \times 2}{7 \times 8\,4} = \frac{3}{28}$

Q)

Q) A and B throw 3 dice. if A throw 14. find B's ~~stands~~ chance of throwing a higher.

To throw a number higher than A. B must throw either 15 or 16 or 17 or 18. ~~Now amount~~

Now a throw ~~one~~ Summing to 18 must be made up of (6, 6, 6) which can occur in 1 way.

17 can be made up of (6, 6, 5) in $\frac{3!}{2!} = 3$ ways;

16 maybe made up of (6, 6, 4) and (6, 5, 5) each of which arrangements can occur in $\frac{3!}{2!} = 3$ ways

15. can be made up of (6, 4, 5) (6, 6, 3) (5, 5, 5) which can occur in 6, 3, 1 ways respectively.

$\therefore$ The no of favourable cases = 3 + 6 + 1 + 3 + 3 + 1 + 3

= 20

In random throw of 3 dice, the exhaustive no of cases = $6^3 = 216$

Hence the required ~~no of~~ probability $= \frac{20}{216} = \frac{5}{54}$

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Among the digits 1, 2, 3, 4, 5, at first one is chosen. and then a second selection is made among the 4 digits. assuming that all 20 possible outcomes have equal probability - find the probability that an odd digit will be selected

(1) for 1st time
(2) for 2nd time
(3) for both time

Ans Total no of cases are $5 \times 4 = 20$

Now
1) There are 12 cases in which 1st digit drawn is odd.

ie 12, 13, 14, 15, 31, 32, 34, 35, 51, 52, 53, 54

So probability of getting 1st digit drawn is odd $= \frac{12}{20} = \frac{3}{5}$

2) Now there are 12 cases in which 2nd digit drawn is odd.

ie (2,1) (3,1), (4,1), (5,1), (1,3) (2,3), (4,3), (5 3) (1,5), (2,5) (3,5) (4,5)

∴ The probability that 2nd digit drawn is odd $= \frac{12}{20} = \frac{3}{5}$.

3) There are 6 cases in which both the digits drawn are odd

ie (1,3) (1,5) (3,1) (3,5) (5,1) (5,3).

∴ The probability that both the digits drawn are odd $= \frac{6}{20}$

$= \frac{3}{10}$.

Statistical (or empirical) probability

(Von Mises definition)

If an experiment is performed repeatly under essentially homogenious and identical conditions, then the limiting value of the ratio of the no of times the event occurs to the no of trials, as the no of trials become indefinitely large, is called the probability of happening of the event, it being assumed that the limit is finite and unique.

Symbollically if in 'n' trials an event 'E' happens 'm' times. Then the probability of happening of E denoted by P(E) is given by

$$P(E) = \lim_{n \to \infty} m/n.$$

Limitations of Emperical Probability.

(1) if an experiment is repeated a large no of times, the experimental conditions may not remain identical or homogeneous.

(2) the limit defined, ie, $\lim_{n \to \infty} m/n$ may not attain a unique value however large n may be

Permutation

Permutation refers to the arrangement which can be made by taking some (say r) of things at a time or all of 'n' things of a time with attention given to the order of arrangement of the selected objects

Suppose we want to arrange 3 students a, b, c by choosing two of them at a time this arrangement can be done in such a way: ab, ac, bc, ba, ca, cb. The arrangement of 3 things taken 2 at a time is denoted as $3P_2$. The arrangements of n objects taking ~~a~~ r at a time is denoted as nP_r.

In general: Suppose there are n objects to be permuted in a row taking all at a time - This can be done in nP_n different ways

$$nP_r = \frac{n!}{(n-r)!}$$

Note

$n! = n(n-1)(n-2)$.

$0! = 1$.

$$nP_r = \frac{n!}{(n-r)!} = n(n-1)\ldots(n-r+1)$$

$$nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$$

$$= n! = n(n-1) \cdot \cdot \cdot \times 3 \times 2 \times 1 .$$

Combination

Combination is a grouping or a selection or a selection of all or a part of given things without reference to there order of arrangements.

if 3 letters a, b, c are given ab, bc, ca are the only combination of the 3 things a, b, c taken two at a time and is denoted as $3C_2$. then other permutation ba, cb, ac are not new combination

$$nC_r = \frac{nP_r}{r!} = \frac{n!/(n-r)!}{r!} = \frac{n!}{(n-r)!\,r!}$$

Notes

$nC_n = 1$

$$nC_0 = \frac{n!}{(n-0)!\,0!} = \frac{n!}{n!} = 1$$

$nC_1 = n$

$nC_r = nC_{n-r}$